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1. Learning Outcomes

After studying this module, you shall be able to:

- Know about the Harrod's model.
- Understand the structure and working of Harrod's model.
- Learn the types of rate of growth for an economy

2. Introduction

Roy Harrod and Evsey Domar worked separately to develop their highly similar models of economic growth and business cycles. The two economists expanded the short-run Keynesian framework to analyze the growth process in the developed economies. Both of them criticized the basic Keynesian framework of income determination in the short run for ignoring the role of investment to create more capacity for the production of output. The investment in physical capital, according to these economists, has a dual role. Dual role of investment here means that investment spending generates income on one hand and also increases the productive capacity of the economy on the other hand. Increase in the income as a result of increase in investment is called the demand side effect while the increase in the productive capacity of economy due to investment is called the supply side effect. Both the economists were interested in finding out an equilibrium growth path which would guarantee a full employment in some sense. Although the two models of Harrod and Domar are similar in many respects but they have some crucial differences as well. Let us investigate the two models below in turns

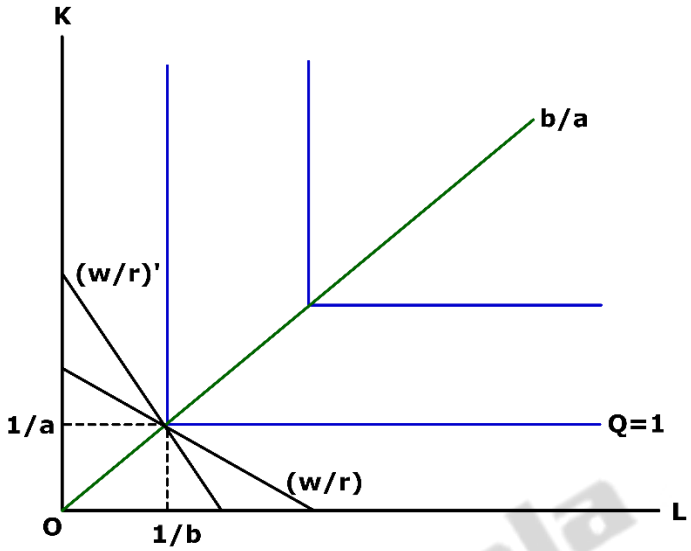
3. Harrod's Model

The model was first given by Harrod in his 1939 paper in the 'Economic Journal'. His first concern was to find that does there exist an equilibrium growth rate of output which if the economy grows at then it will continue to grow at the same rate moving over time?His second concern was to investigate that whether such an equilibrium growth path is stable in the sense that if ever the economy grew at some different rate then would it automatically move towards equilibrium growth rate in due time?

Basic assumptions of the Harrod's Model

1. Savings and investment refer to income of the same period. Both Saving and Investment are net, i.e. over and above the depreciation.

2. The economy saves a constant proportion of its income, which implies that the marginal propensity to save (MPS) i.e. $\frac{dS}{dY}$ is equal to average propensity to save (APS) i.e. $\frac{S}{Y}$. Since if MPS ≠ APS, then the latter could not stay constant.
3. Income is determined by investment through the multiplier process while investment is determined through the process of accelerator.
4. If plans of investment are realized then the firms don't change the rate of desired investment whereas if plans are under realized i.e. actual investment is less than planned investment or over realized i.e. actual investment is more than planned then firms increase or decrease respectively the rate of desired investment.
5. The economy is assumed to begin with full employment of capital.
6. There are no lags in the adjustment between demand and supply, especially between investment and creation of productive capacity.
7. Aggregate output in the economy can be written as a function of aggregate physical capital and aggregate labor measured in suitable units respectively i.e. $Y = F(K, L)$. It is further assumed that there are constant returns to scale in the aggregate production function which means that if both the factors are changed by some equal proportion then output also gets by same proportion.
8. General Price level and the rate of interest remains fixed. It means that the relative prices of capital and labor remain constant as the economy grows. This assumption has a very crucial implication for the Harrod model. A constant relative factor price implies a constant capital-labor ratio i.e. $\frac{K}{L}$ in the economy over time. Some authors like Branson formulate the requirement of fixed $\frac{K}{L}$ in the Harrod model using an L-shaped fixed proportion production function of the form $Y = \text{Min}(aK, bL)$as shown below.



If the production function is conceived to be of a perfectly complementary form as above then a change in relative factor prices from say (w/r) to (w/r)'will not change the ratio $\frac{b}{a}$ in which the factors are used. Although there is no need to assume such a functional form of aggregate production provided we assume constant relative price of factors.

9. The implication of constant returns to scale along with fixed proportion of factors implies constant capital-output ratio as well as constant labor-output ratio overtime. The constancy of capital-output ratio is a very important assumption of the Harrod's model.
10. There is no role of government in influencing aggregate demand and supply.

4. Structure and working of the Harrod's Model

4 Harrod wanted to find out that rate of growth of investment or output which will sustain itself overtime. In order to find that out Harrod does the marriage of multiplier and accelerator to arrive at his most fundamental growth equation. Keynesian multiplier can be written says that

$\Delta Y = \frac{\Delta I}{s}$ (1)

Here Y stands for the aggregate output, I for net investment and s for the savings ratio in the economy.

The accelerator theory of investment tells us that the net investment planned or desired in any period in an economy is a fixed multiple of the expected change in output during that period i.e.

$I = C_r \Delta Y$ (2)

Here $C_r = \frac{\Delta K_r}{\Delta Y}$ stands for the desired change in capital stock per unit increment in output. C_r is also known as desired incremental capital-output ratio. If we combine equations 1 and 2 above by eliminating ΔY then we get that rate of growth of planned investment

$\frac{\Delta I}{I} = \frac{s}{C_r}$ (3)

We know that in any short run equilibrium of the economy, planned net savings must equal planned net investment i.e. $I = S$ which implies that in a short run equilibrium $I = S = sY$ or $I/Y = s$. In other words, to keep their ratio fixed, planned investment and output must grow at the same rate i.e. $\frac{\Delta I}{I} = \frac{\Delta Y}{Y}$ if the economy has to always remain in short run equilibrium at all points in time while growing. Substituting $\frac{\Delta Y}{Y}$ in place of $\frac{\Delta I}{I}$ in equation 3 above, we get

$\frac{\Delta Y}{Y} = \frac{s}{C_r}$ (4)

Equation 4 is the most fundamental equation in Harrod's model since it says that if economy is growing at the rate given in the right hand side of equation 4, then planned investment is always equal to planned savings along the moving growth path. To see why this happens we can rewrite the equation 4 as $C_r \Delta Y = sY$ where the right hand side is nothing but the planned savings and the left hand side is nothing but the planned investment. Since we generally assume in Keynesian framework that savings are always realized and by truism we know that actual savings must always equal actual investment therefore equation 4 can be understood to give us that growth path where planned investment must equal actual investment. The assumption number 7 above tells us that if the plans of firms are met, then there is no reason for them to change their rate of investment which through multiplier would ensure an unchanged rate of growth. Therefore if the economy grows at the rate $\frac{s}{C_r}$ then there is no force to deviate it from this constant growth path. Such an equilibrium growth rate is defined as Warranted (Required) rate of growth by Harrod and denoted as g_w. In his own words, it is "that rate of growth which, if occurs, will leave all the parties satisfied that they have produced neither more nor less than the right amount. … It will put them into a frame of mind which will cause them to give such orders as will maintain the same rate of growth". Therefore we may write

$g_w = \frac{s}{C_r}$ (5)

In fact he defines the following three different types of rate of growth for an economy.

1. Warranted (Required) rate of growth denoted as g_w.
2. Actual rate of growth denoted as g_a.
3. Natural rate of growth denoted as g_n.

Using a similar terminology as in case of warranted rate of growth we may write that

$$g_a = \frac{\Delta Y}{Y} = \left(\frac{\Delta Y}{I} \times \frac{I}{Y}\right) = \left(\frac{\Delta Y}{\Delta K} \times \frac{I}{Y}\right) = \frac{s}{C} \qquad (6)$$

Here S is the saving ratio and C is the actual (not desired) ratio of actual change in capital i.e. actual net investment to the change in output i.e. $C = \frac{\Delta K}{\Delta Y} = \frac{I}{\Delta Y}$. Thus C is the actual incremental capital-output ratio.

Natural rate of growth is defined by Domar as that rate of growth of output which is required to fully employ the entire growing labor force. Since labor-output ratio is assumed to be constant, therefore the natural rate of growth of output must be equal to the rate of growth of labor. Suppose if the rate of growth of population is given by n then the natural rate of growth must be equal to n.

$$g_n = n \qquad (7)$$

You may think of growth of labor force as not just an increase in number of labor but an increase in number of effective labor to incorporate the increases in labor productivity.

We know that in the short run disequilibrium occurs whenever actual investment is not equal to planned investment i.e. there is either an unplanned positive or an unplanned negative addition to stock of inventories. This equilibrium is restored by the firms in subsequent periods through increase or decrease in output in case of unplanned removal or unplanned addition to inventories respectively. Does a similar response mechanism also bring the economy back to required equilibrium growth path in case of any deviation from it? After deriving the warranted or equilibrium rate of growth, Harrod's next concern was to check if the equilibrium is stable i.e. is any deviation from the equilibrium path self-correcting over time?

Amartya Sen in his introduction to Harrod in the book 'Growth Economics' has formulated the following adaptive expectation response model of the firms wherein

$$I_t = C_r\,(Y_t^e - Y_{t-1})$$
$$g_t^e - g_{t-1}^e = \lambda(g_{t-1} - g_{t-1}^e)$$

The first equation says the investment in any period is a function of the increase in output expected in this period over the previous period's output. The second equation says that the expected rate of growth of output increases by a constant positive multiple of the

difference between actual and expected rate of growth of output in the previous observed period. For example, if actual output in previous period was more than expected output i.e. actual growth was more than expected growth in previous period then the firms will revise their expectations of growth of output upwards in the current period implying a larger expected output in the current period and therefore a larger investment given by the first equation of Sen. The reverse will hold true whenever actual will be less than expected.

Let us consider the following two scenarios:

1. $g_a > g_w$
2. $g_a < g_w$

Using equations 5 and 6 above, we may write that if

$$g_a > g_w \Longrightarrow C < C_r \Longrightarrow \Delta K < \Delta K_r \Longrightarrow rate\ of\ palnned\ I\ is\ increased$$
$$\Longrightarrow\ g_a\ increases$$

$$g_a < g_w \Longrightarrow C > C_r \Longrightarrow \Delta K > \Delta K_r \Longrightarrow rate\ of\ planned\ I\ is\ decreased$$
$$\Longrightarrow\ g_a\ decreases$$

The first implication above considers the situation when for some reason the actual rate of growth in the economy happens to be more than the warranted rate of growth. Under such a situation, the actual incremental capital per unit of incremental output will be less than the desired incremental capital per unit of incremental output. A shortfall in actual capital vis-à-vis planned capital will take either the form of unplanned shortfall in the stock of inventories or an excess demand for the equipment. Both the situations will be inflationary and responded by the firms through increase in the rate of planned investment according to assumption 4. Such an increase will cause a further increase in the actual rate of growth of output through the multiplier mechanism. It means that whenever the actual growth rate of output is more than the warranted rate of growth, the response by the firms will make the actual growth rate even larger than the warranted growth rate. Therefore the response mechanism which worked to create equilibrium in the short run will create further disequilibrium in the long run and take the economy further away from the equilibrium growth path towards a boom.

The second implication above considers the situation when the actual rate of growth in the economy happens to be less than the warranted rate of growth. Under such a situation, the actual incremental capital per unit of incremental output will be more than the desired incremental capital per unit of incremental output. A shortfall in actual capital vis-à-vis planned capital will again take either the form of unplanned addition to the stock of

inventories or an excess capacity of the equipment. Both the situations will be deflationary and responded by the firms through decrease in the rate of planned investment according to assumption 4. Such a decrease will cause a further decrease in the actual rate of growth of output through the multiplier. It means that whenever the actual growth rate of output is less than the warranted rate of growth, the response by the firms will make the actual growth rate even smaller than the warranted growth rate. Therefore the response mechanism will create further disequilibrium in the long run and take the economy further away from the equilibrium growth path towards a depression. The above analysis concludes that the equilibrium growth path of Harrod is completely unstable and any deviation is further aggravating rather than self-correcting. For this reason, the equilibrium growth is said to have a Knife-Edge Stability. If you make a slip on either side, you keep falling.

The instability of equilibrium in the Harrod model was used by him to explain the business cycles above and below the trend path of warranted growth. But we know that business cycles always have peaks and troughs. Harrod explained these through the use of natural rate of growth of output. Remember from equation 7 that $g_n = n$. Harrod argues that in an economy with constant capital-labor ratio, the actual growth rate of output can never be more than the natural rate of growth of output i.e. $g_a \leq g_n = n$. In other words, growth is constrained by labor.

Here again we consider two possible scenarios:

1. $g_a \leq g_n < g_w$
2. $g_n > g_w$

In the first situation, natural rate is less than the warranted rate which implies that the actual rate which has to be less than or equal to the former will also be less than the warranted rate. Such a situation as we have seen above will push the actual rate further below and bring a deflationary gap in the economy. During this gap, neither the labor nor the capital will be fully employed. The less than full employment of capital is obvious since at all rates of growth below the warranted, actual capital stock must be more than planned capital stock. Correction will only be possible if the warranted rate of growth falls below to become equal to the actual. A fall in warranted rate requires a fall in savings or an increase in C_r. But an increase in latter is not profitable and thus possible under conditions of deflation. In fact, C_r is bound to fall. But Harrod argues that the savings rate in the economy will fall during the deflation due to redistribution in income against the capitalists and will fall more in proportion to fall in C_r thereby decreasing the warranted rate. When it has

fallen enough, then actual will once again become greater than warranted and begin to recover thereafter.

In the second situation, natural rate is more than the warranted rate. In this situation, if actual is equal to warranted then there the economy will move along a constant growth path where there will full employment but persistent growing unemployment of labor. However, if actual is more than warranted, then actual rate will keep growing till it reaches its bounds of natural rate of growth. Although the unemployment will keep o reducing but the capital will be in excess demand putting inflationary pressures. But this rate high rate of growth will not be sustained forever since due to inflation there will be a redistribution of income in favor of capitalists implying an increase in savings rate faster than the increase in desired incremental capital output ratio. On the other hand, if actual is less than warranted, then the economy will move further in depression with high labor unemployment levels and also unused capacity. This will keep happening unless the warranted rate falls below through the mechanism discussed in the above paragraph. Note that although warranted growth rate changes, but the warranted growth path does not which requires that actual rate of growth must be equal to warranted rate of growth.

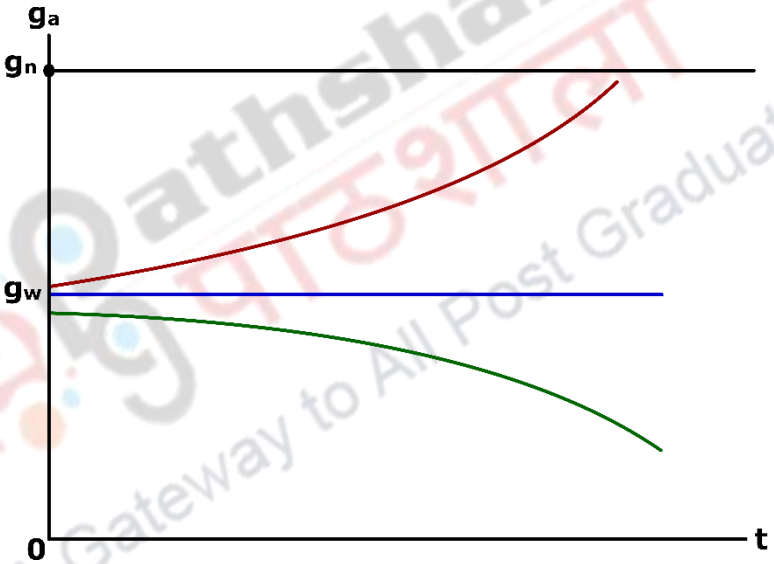
The only situation in the Harrod's model where there is full employment of both capital and labor under equilibrium is when

$$g_a = g_w = g_n$$

~~This is so improbable a situation that Mrs. Joan Robinson has termed it 'Golden Age'4.5 Other factors~~

5. Summary

1. The equilibrium rate of growth which maintain itself is called the warranted rate of growth and given by $g_w = \frac{s}{C_r}$
2. Along this warranted growth path, there is full employment of capital and firms have all the satisfaction to continue investing at a rate that the same growth rate is achieved.
3. The equilibrium is unstable and any diversion from this growth path will result in further aggravation of the gap between actual and warranted growth rate. The figure below shows the time path of the growth rate of an economy under three possible situations.



In this figure we assume that natural rate is more than the warranted rate.

4. Only if $g_a = g_w = g_n$, there is full employment of both labor and capital under equilibrium growth path.